## MEASUREMENT OF THE COMPLEX COMPLIANCE OF COATINGS OF ELASTIC MATERIALS

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A procedure for measuring the compliance of viscoelastic coatings that are used to reduce the hydrodynamic friction has been developed (the frequency range of measurements is from 400 Hz to 2 kHz). The set of force transducers differing by the diameter of the surface of contact with a coating has been manufactured. The frequency and amplitude characteristics of the transducers have been given. It has been shown that the dynamic compliance is independent of the static deformation  $\varepsilon$  if  $\varepsilon > 3\%$ . The diameter of the coating sample for which the edge effects are of little importance has been selected. It has been shown that with a twofold increase in the coating the measured resonant frequency of the coating decreases twofold as well.

Experiments [1–4] have convincingly shown a reduction in the friction in the case of turbulent flow above a compliant wall. Deformation of the surface of a compliant coating under the action of turbulent pressure pulsations is the initial cause of this reduction [5]. In the existing theories [6, 7] which explain the mechanism of this phenomenon, one assumes the linear dependence of the deformation on pressure. The proportionality coefficient is called the compliance of a coating (the reciprocal of it is rigidity). Another important parameter is the phase angle by which the deformation lags behind the pressure. A method of direct measurement of these quantities has been proposed and tested in [8]. It was developed further in the present work.

A diagram of the setup is shown in Fig. 1. The electromagnetic stand produces harmonic vibrations in the vertical direction in the band of frequencies 60 Hz–10 kHz. The vibroacceleration amplitude is measured by the standard piezoceramic accelerometer 1. The coating is fixed on the massive plate 4 located above a vibrating table and can be displaced vertically. The plate is installed on the stationary casing of a vibrator which is protected by vibroinsulation spacers against the vibrations of the base. The coating is smoothly brought into contact with transducer 2 measuring the dynamic force. A device for measuring vertical displacement enables us to set different initial static deformations.

To measure the dynamic force use is made of a set of transducers with different diameters of the contact surface (from 2 to 25 mm). A thin piezoceramic plate cemented with a conducting cement to the upper plane of the transducer is the working element. Since for the same power of the vibrator the amplitude of displacement is in inverse proportion to the frequency squared, the force acting on the balance decreases in the same manner as the frequency increases. To extend the measurement range toward high frequencies we use a band-pass filter cutting off frequencies lower than 250 Hz and higher than 10 kHz. For this purpose we employed Butterworth filters of the Robotron acoustic complex; their characteristics are shown in Fig. 2. To match the high resistance of the transducers with the input resistance of loads use is made of U1 and U2 preamplifiers with an input resistance of  $\approx$ 40 M $\Omega$  and an amplification factor of 10.

The sensitivity of the force transducers was determined as follows. The transducer was brought out of contact with a compliant coating and was loaded by the inert mass m, as is shown in Fig. 3a. The produced signal  $U_{\text{sign}} = U_{\text{sign}} \exp [i(\Theta_{\text{sign}} - \Theta_{\text{f}})]$  consists of two parts: the signal of the transducer without a load mass  $U_0 = U_0 \exp [i(\Theta_0 - \Theta_{\text{f}})]$  and the signal caused by the load mass.

Thus,

$$\hat{U}_{\text{sign}} = \hat{U}_0 + k_{\text{tr}} P , \qquad (1)$$

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Fig. 1. Diagram of the setup: 1) vibroacceleration transducer (accelerometer); 2) force transducer; 3) sample; 4) movable holder of a sample.



Fig. 2. Characteristics of the filter: a) amplification factor; b) phase shift. f, kHz;  $\Theta$ , angular degrees.

where P = ma/S is the dynamic pressure exerted on the transducer by the mass. Here S is the area of contact of the transducer with the inert mass (or with the compliant coating) and the value of the vibroacceleration is determined by the signal of the reference transducer:  $a = k_{ref}U_{ref}$ .

As a result we obtain

$$k_{\rm tr} = \frac{\hat{U}_{\rm sign} - \hat{U}_0}{U_{\rm ref}} \frac{S}{mk_{\rm ref}} = |k_{\rm tr}| \exp(i\Theta_{\rm tr}),$$

where

$$|k_{tr}| = \frac{S}{mk_{ref}} \frac{U_{sign}}{U_{ref}} \left\{ \left[ \cos \left(\Theta_{sign} - \Theta_{f}\right) - \frac{U_{0}}{U_{sign}} \cos \left(\Theta_{0} - \Theta_{f}\right) \right]^{2} + \left[ \sin \left(\Theta_{sign} - \Theta_{f}\right) - \frac{U_{0}}{U_{sign}} \sin \left(\Theta_{0} - \Theta_{f}\right) \right]^{2} \right\}^{1/2};$$

$$\Theta_{tr} = \arctan \frac{\sin \left(\Theta_{sign} - \Theta_{f}\right) - \frac{U_{0}}{U_{sign}} \sin \left(\Theta_{0} - \Theta_{f}\right)}{\cos \left(\Theta_{sign} - \Theta_{f}\right) - \frac{U_{0}}{U_{sign}} \cos \left(\Theta_{0} - \Theta_{f}\right)}.$$
(2)



Fig. 3. Schemes of measurement: a) calibration; b) determination of the compliance.



mV/Pa; f, kHz.

Figure 4a shows the frequency characteristic of the transducer with the diameter of the working surface d = 10 mm for different load masses. It is seen that the sensitivity of the transducer is independent, in practice, of the frequency in the working range of frequencies from 400 Hz to 2 kHz. In Fig. 4b, the slope of the curve is the sensitivity of the transducer. Good linearity of the transducer relative to the applied load is noted. The rotation of the phase by the transducer turned out to be zero, in practice (which was to be expected because of the noninertia of the transducer at these frequencies).

The compliance of the coating is determined analogously (Fig. 3b). By definition, the compliance of a coating  $\stackrel{\wedge}{C} = |C| \exp(i\Theta)$  is equal to the ratio of the displacement z to the applied pressure P.

The value of the displacement is determined by the reference transducer of vibroacceleration:

$$z = -\frac{a}{\omega^2} = -\frac{k_{\text{ref}}}{\omega^2} U_{\text{ref}}.$$

The reaction of the coating to this displacement is found from formula (1):

$$P = \frac{U_{\text{sign}} \exp \left[i \left(\Theta_{\text{sign}} - \Theta_{\text{f}}\right)\right] - U_0 \exp \left[i \left(\Theta_0 - \Theta_{\text{f}}\right)\right]}{k_{\text{tr}}}$$

Finally we have

$$|C| = \frac{k_{\text{ref}}k_{\text{tr}}}{\omega^2} \frac{U_{\text{ref}}}{U_{\text{sign}}} \left[ \cos\left(\Theta_{\text{sign}} - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 \right]^2 + \frac{U_0}{U_{\text{sign}}} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 \right]^2 + \frac{U_0}{U_0} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_0} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_0} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_0} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_0} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_0} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_0} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_0} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_0} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_0} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) - \frac{U_0}{U_0} \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_0} \left[ \cos\left(\Theta_0 - \Theta_{\text{f}}\right) \right]^2 + \frac{U_0}{U_$$



Fig. 5. Voltage of the signal vs. initial compression of the sample: 1) f = 400, 2) 600, and 3) 800 Hz.  $U_{\text{sign}}$ , mV;  $\varepsilon_{\text{stat}}$ , %.



Fig. 6. Comparison of the compliance measured for different diameters of the sample (a) [1) D = 100 and 2) 200 mm] and for different thickness (b) [1) H = 5 and 2) 10 mm]. |C|, m/Pa; f, kHz.

+ 
$$\left[\sin \left(\Theta_{\text{sign}} - \Theta_{\text{f}}\right) - \frac{U_0}{U_{\text{sign}}}\sin \left(\Theta_0 - \Theta_{\text{f}}\right)\right]^2\right]^{1/2}$$

Formula (2) also determined the phase shift  $\Theta$ .

Calculation of the sensitivity of the transducers and computation of the compliance of the coatings are made much easier when the sheets of Excel tables are used.

The procedure was checked on coatings manufactured from SKTN polydimethylsiloxane [9]. We manufactured three samples: of diameter D = 100 mm and thickness H = 5 mm, of D = 100 mm and H = 10 mm, and of D = 200 mm and H = 5 mm.

First we determined the required value of the initial static displacement. The point of contact of the transducer with the coating was determined from the beginning of the growth of the amplitude of a signal from transducer 2 on the oscilloscope screen. The displacement is measured with an error of  $\pm 2 \,\mu$ m by the optical unit of an IZA2 comparator. Figure 5 shows a typical dependence. It is seen that the value of the signal is related to the initial deformation  $\varepsilon_{\text{stat}} = \Delta H/H$  to a much lesser extent if  $\varepsilon_{\text{stat}} > 3\%$ .

Figure 6a compares the coatings of the same thickness but of different diameter. The compliance was measured at the center of the sample. The value of the dynamic deformation is  $\varepsilon_{dyn} = \sqrt{z^2}/H \approx 0.02\%$  for  $\varepsilon_{stat} = 3\%$ .

Good coincidence of the results shows that the selected diameter of the coating is sufficient for the edge effects to be of little importance in propagation of deformation waves.

Figure 6b compares the coatings of the same diameter but of different thickness. With a twofold increase in the thickness the resonant frequency of the coating decreased twofold as well (from  $\approx$ 1400 Hz to  $\approx$ 700 Hz). This agrees with the theory since at resonance the wavelength is equal to the coating thickness when the diagram of deformations over the coating thickness is considered. The compliance increased nearly twofold, since it is in proportion to the coating thickness.

## NOTATION

*a*, vibroacceleration, m/sec<sup>2</sup>; *C*, compliance, m/Pa; *d* and  $S = \pi d^2$ , diameter and area of the contact surface, m and m<sup>2</sup>; *H* and *D*, thickness and diameter of the sample, m; *K*, transmission coefficient of the filter;  $k_{tr}$ , sensitivity of the force transducer, mV/Pa;  $k_{ref}$ , sensitivity of the vibroacceleration transducer (accelerometer), m·sec<sup>-2</sup>/mV; *m*, calibration mass, g; *U*, voltage, mV; *f* and  $\omega = 2\pi f$ , frequency and cyclic frequency;  $\varepsilon$ , relative deformation;  $\Theta$ , phase shift. Superscript:  $^{\wedge}$ , complex quantity. Subscripts: tr, force transducer; ref, reference vibroacceleration transducer; sign, signal; 0, without a load; f, filter; stat, static; dyn, dynamic.

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